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Glauber approximation in inelastic e-H scattering

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Glauber approximation has been applied to calculate the total and the differential cross sections for $1s$–$2s$ and $1s$–$2p$ excitations in e-H scattering. In the intermediate energy region, our results for the total cross section of $1s$–$2p$ excitation are in better agreement with the experimental observations than other existing theoretical results. In the case of $1s$–$2p$ excitation the total cross section curve almost coincides with the experimental findings. The differential cross sections for both the cases are more sharply peaked in the forward direction than those in Born approximation.

Introduction

With the recent developments in the experimental techniques, considerable theoretical interests have been focussed upon electron-atom collisions. Electron-hydrogen system is theoretically the simplest one and as such has been most extensively studied. In the case of inelastic electron-hydrogen (e-H) collision process, there are long standing marked differences between the experimental results and the theoretical findings. There was no appreciable improvement in the theoretical results in spite of repeated attempts. Akerib & Borowitz (1961) have applied the impulse approximation to the inelastic e-H scattering. A new method, which explicitly takes into account the repulsion between the atomic and incident electrons in the choice of the total wave function, has been introduced by Vainshtein, Presnyakov & Sobelman (1963). Ochkur (1963) has given a modified form of Born-Oppenheimer approximation allowing for exchange interaction. Recently (1969), he has presented an improved version of this previous work. Sloan & Moore (1968) have given a theoretical formulation for both the elastic and inelastic processes based on Foddeev equation (1961) for three particle scattering. This approximation amounts to an unitarized Born approximation with the exchange effect taken into account. Several workers (Damburg & Peterkof 1962, Burke, Schey & Smith 1963) have applied the close coupling approximation to the e-H scattering problem. This approximation, though theoretically sound, is laborious in practice and the results obtained in the case of the inelastic processes are not upto the expectation.

The purpose of the present paper is to make an analysis of the inelastic ($1s$–$2s$ and $1s$–$2p$) electron-hydrogen scattering processes. The dynamical basis of our calculation is the multiple scattering model proposed by Glauber (1959). This approximation is extensively used in nuclear and particle physics Bessel

& Wilkin 1968, Harrington 1968). In atomic physics, it has been applied to elastic electron-hydrogen scattering (Franco 1968, Tai *et al* 1969). Glauber approximation is based on Eikonal approximation. The latter applies to the scattering by a fixed potential and is thereby restricted to single scattering whereas Glauber approximation takes account of multiple scattering. Contrary to the method of impulse approximation the interaction between the incident electron and the proton has been taken into account in the present method.

We have calculated the $1s$–$2s$ and $1s$–$2p$ excitation cross sections in e-H collision covering the energy region 10·6 eV to 200 eV.

Theory

We consider the target proton to be infinitely heavy and the origin of the co-ordinate to be placed at the position of the proton. Let $\boldsymbol{r}$ denote the position vector of the atomic electron and $\boldsymbol{b}$ be the impact parameter vector relative to the origin In Glauber approximation, the amplitude of scattering $F_{fi}(\alpha)$ for the process in which the hydrogen atom undergoes a transition from an initial state i with wave function ϕ_i to a final state f with wave function ϕ_f is given by (Franco 1968)

$$F_{fi}(\alpha) = \frac{ik_0}{2\pi} \int \phi_f^*(\boldsymbol{r})\Gamma(\boldsymbol{b}, \boldsymbol{r})\phi_i(\boldsymbol{r}) \exp(i\boldsymbol{q}.\boldsymbol{b})d^2b\, d\boldsymbol{r}, \tag{1}$$

where $\boldsymbol{q} = \boldsymbol{k}_0 - \boldsymbol{k}_1$, $\boldsymbol{k}_0$ and $\boldsymbol{k}_1$ being, respectively, the momenta of the incident and scattered electron The double integration with respect to d^2b is over the plane perpendicular to the incident beam direction. $\Gamma(\boldsymbol{b}\ \boldsymbol{r})$ has the form (Glauber 1959)

$$\Gamma(\boldsymbol{b}, \boldsymbol{r}) = 1 - \exp(2i\chi(\boldsymbol{b})), \qquad \dots (2)$$

where $\chi(\boldsymbol{b})$ is the phase shift corresponding to the impact parameter $\boldsymbol{b}$. According to Glauber, the phase shift due to a number of scattering centres is just the sum of the individual phase shifts due to each, taken separately, of course, at the appropriate values of the impact parameters. Thus for the case of e-H scattering we may write (Franco 1968)

$$\Gamma(\boldsymbol{b}, \boldsymbol{r}) = 1 - \exp\left[\left(-\frac{iZe^2}{v}\right)\int_{-\infty}^{\infty}\{(b^2+\zeta^2)^{-\frac{1}{2}} - [(\boldsymbol{b}-\boldsymbol{s})^2+(\zeta-Z)^2]^{-\frac{1}{2}}\}d\zeta\right]$$

$$= 1 - \left(\frac{|\boldsymbol{b}-\boldsymbol{s}|}{b}\right)^{2in}$$

with $\boldsymbol{r} = \boldsymbol{s} + \boldsymbol{z}$, where $\boldsymbol{s}$ is the component of $\boldsymbol{r}$ perpendicular to the incident beam and $n = e^2/\hbar v$, v being the velocity of the incident electron.

Next we calculate the scattering amplitudes for the two different cases under consideration.

(A) $(1s\text{–}2s)$ *case* :

Here the initial and final states are, respectively, the $1s$ and $2s$ states of hydrogen atom;

i.e., $\phi_{1s}(r) = (\pi a_0^3)^{-\frac{1}{2}} \exp(-r/a_0)$ and $\phi_{2s}(r) = (2^3 a_0^3 \pi)^{-\frac{1}{2}}\left(1 - \dfrac{r}{2a_0}\right)\exp(-r/2a_0)$, where a_0 is the Bohr radius.

We take $\boldsymbol{q}$ to be perpendicular to the incident momentum $\boldsymbol{k}_0$ (figure 1), this assumption (Bessel & Wilkin 1968) is justified in Glauber's model which is applicable to small angle scattering at high energy.

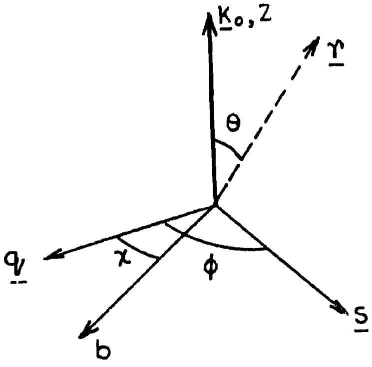

FIGURE 1. Coordinate system used for inelastic e-H scattering.

Substituting the expressions for $\phi_{1s}(r)$, $\phi_{2s}(r)$ and $\Gamma(\boldsymbol{b}, \boldsymbol{r})$ and changing integration variables from ϕ, χ to $\phi'\,(=\phi-\chi)$, χ we can write equation (1) as

$$F_{fi}(\alpha) = \frac{ik_0}{2^{5/2}\pi^2 a_0^3} \int \left(1 - \frac{\sqrt{s^2+z^2}}{2a_0}\right)\exp\left(-\frac{3}{2a_0}\sqrt{s^2+z^2}\right)$$

$$\left[1 - \left(\frac{b^2+s^2-2bs\cos\phi'}{b^2}\right)^{i\eta}\right]\exp(iqb\cos\chi)\,bs\,db\,d\chi\,dz\,ds\,d\phi'$$

Integrating with respect to χ we have

$$F_{fi}(\alpha) = \frac{ik_0}{2^{3/2}\pi a_0^3} \int \left(1-\frac{\sqrt{s^2+z^2}}{2a_0}\right) \exp\left(-\frac{3}{2a_0}\sqrt{s^2+z^2}\right)$$

$$\left[1-\left(\frac{b^2+s^2-2bs\cos\phi'}{b^2}\right)^{in}\right] J_0(qb)\times bs\, db\, dz\, ds\, d\phi'$$

where the integration with respect to b is over the interval $(0, \infty)$. We perform the angular integration with respect to ϕ' and obtain

$$F_{fi}(\alpha) = \frac{ik_0}{\sqrt{2}a_0^3} \int \left(1-\frac{\sqrt{s^2+z^2}}{2a_0}\right) \exp\left(-\frac{3}{2a_0}\sqrt{s^2+z^2}\right)$$

$$\left[1-\left(\frac{2s}{b}\right)^{in} G(y)\right] J_0(q\,b) bs\, dz\, db\, ds,$$

where $G(y) = y^{-in}(1-y^2)^{\frac{1}{2}+in}\,{}_2F_1(\frac{1}{2}+\frac{1}{2}in,\ 1+\frac{1}{2}in,\ 1,\ y^2)$ and $y = \frac{2bs}{b^2+s^2}$ · ${}_2F_1$ being the hypergeometric function. Now we perform the integration with respect to $z(-\infty, \infty)$ and get

$$F_{fi}(\alpha) = \frac{2^n ik_0}{\sqrt{2}a_0^3} \int \left[\frac{4}{3}sK_1\left(\frac{3}{2a_0}s\right) - \frac{s^2}{2a_0}K_2\left(\frac{3}{2a_0}s\right)\right] \times \left[1-\left(\frac{2s}{b}\right)^{in} G(y)\right],$$

$$\times J_0(q\,b) bs\, db\, ds \qquad \ldots (4)$$

where K_1 and K_2 are the Bessel functions of the third kind. The integration with respect to s is over the interval $(0, \infty)$ Introducing the polar variables r, θ given by $b = r\cos\theta$ and $s = r\sin\theta$. to the integral (4) and evaluating the radial integration we have

$$F_{fi}(\alpha) = -\frac{2^{11}ik_0 a}{3^6\times\sqrt{2}} \int_0^{\pi/2} \sin^3\theta\cos\theta\left[\sin^4\theta - \frac{28}{9}(a_0 q)^2\cos^2\theta\sin^2\theta + \frac{64}{81}(a_0 q)^4\cos^4\theta\right]$$

$$\times\left(\sin^2\theta + \frac{4}{9}(a_0 q)^2\cos^2\theta\right)^{-5}$$

$$\times[1-(|\cos 2\theta|/\cos\theta)^{2in}|\cos 2\theta|\,{}_2F_1(\tfrac{1}{2}+\tfrac{1}{2}in,\ \tfrac{1}{2}in+1,\ 1;\ \sin^2 2\theta)]d\theta \qquad \ldots (5)$$

(B) ($1s$–$2p$) *case*.

Here the possible final states, with k_0 as the polar axis, are

$$\phi_{2p,\pm 1}(r) = (2^6\pi a_0^5)^{-\frac{1}{2}} r\exp(-r/2a_0)\sin\theta\exp(\pm i\phi)$$

$$\phi_{2p_0}(r) = (2^5\pi a_0^5)^{-\frac{1}{2}} r\exp(-r/2a_0)\cos\theta$$

One can easily find that the factor $r\cos\theta$ that appears in the wave function for $2p0$. makes the corresponding matrix elements vanish. Further, it can be shown that the cross sections for the other two states are the same. Therefore we need calculate only a single scattering amplitude. As in the previous case we now substitute the expressions for $\phi_{1s}(r)$, $\phi_{2p,-1}(r)$ and $\Gamma(\boldsymbol{b}, \boldsymbol{r})$ in equation (1) and obtain

$$F_{fi}(\alpha) = \frac{ik_0}{2^4\pi^2 a_0^4} \int s \exp\left(-\frac{3}{2a_0}\sqrt{s^2+z^2}\right)\left[1-\left(\frac{b^2+s^2-2bs\cos\phi'}{b^2}\right)^{in}\right]$$

$$\exp(iq\, b\cos\chi + i\chi)\exp(i\phi')\, bs\, db\, d\chi\, dz\, ds\, d\phi'$$

Integrating with respect to χ we obtain

$$F_{fi}(\alpha) = \frac{-k_0}{2^3\pi a_0^4} \int s^2 \exp\left(-\frac{3}{2a_0}\sqrt{s^2+z^2}\right)\left[1-\left(\frac{b^2+s^2-2bs\cos\phi'}{b^2}\right)^{in}\right]$$

$$\times J_1(qb) b\, db\, dz\, ds\, d\phi'$$

Performing the integration with respect to ϕ' we get

$$F_{fi}(\alpha) = \frac{-ink_0}{2^3 a_0^4} \int s^2 \exp\left(-\frac{3}{2a_0}\sqrt{s^2+z^2}\right)\left(\frac{2s}{b}\right)^{in} G(y) J_1(qb) b\, db\, ds\, dz$$

where $$G(y) = y^{1-in}{}_2F_1\left(\tfrac{1}{2}-\tfrac{1}{2}in,\ 1-\tfrac{1}{2}in,\ 2;\ y^2\right)$$

and $$y = \frac{2bs}{b^2+s^2}$$

Following the same procedure as in $1s$–$2s$, we finally get the scattering amplitude as

$$F_{fi}(\alpha) = \frac{-i2^{12}nk_0a_0^2}{3^6}\int_0^{\pi/2}(a_0q)\sin^5\theta\cos^3\theta\,(\cos\theta)^{-2in}$$

$$\times\left(\sin^2\theta - \frac{4}{9}(a_0q)^2\cos^2\theta\right)\left(\sin^2\theta+\frac{4}{9}(a_0q)^2\cos^2\theta\right)^{-5}$$

$$\times\, {}_2F_1\left(\tfrac{1}{2}-\tfrac{1}{2}in,\ 1-\tfrac{1}{2}in,\ 2;\ \sin^2 2\theta\right)d\theta \qquad \ldots (6)$$

The differential cross-section for a particular transition is obtained by the relation

$$I(\alpha) = \frac{k_f}{k_i}\,|F_{fi}(\alpha)|^2 \qquad \ldots (7)$$

The total cross section for the process is given by

$$Q = 2\pi\int_0^{\pi}[F(\alpha)]\sin\alpha\, d\alpha \qquad \ldots (9)$$

Results and Discussion

We have calculated the differential cross sections for $1s$–$2s$ and $1s$–$2p$ excitations with the help of the equations (5), (6) and (7). The integration in equations (5) and (6) have been done numerically using a 16-point Gaussian quadrature. In figure 2, we have compared our results for the differential cross sections for $1s$–$2s$ excitation at incident energies 50, 100, and 200 eV with the corresponding results of the first Born approximation (FBA). Table 1 furnishes

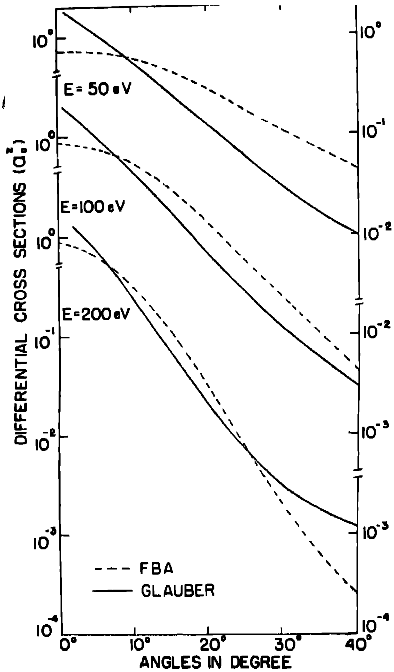

Figure 2. Excitation differential cross-sections of hydrogen $2s$ level in the first Born and Glauber approximations.

Table 1. Differential cross-sections per unit solid angle in units of $a_0{}^2$ per steradian for excitation of $2p$ level of hydrogen.

Electron Energy (eV)	Cosine of angle of scattering					
	0.9999	0.990	0.985	0.939	0.868	0.750
50	32.06	20 99	7 56	0.94	0.17	0.03
100	83.51	22 56	4.22	0.25	0.03	0.004
200	259.30	12.70	1.44	0.03	0.003	0.0004

the calculated values of the differential cross-section for $1s$–$2p$ excitation at different values of the cosine of the angle of scattering for these incident energies.

In the calculation of the total cross-section we have carried out the integration with respect to the cosine of the angle of scattering numerically. Depending upon the nature of the integrand we have divided the total range of integration (−1, 1) into suitable sub-intervals. In each of the sub-intervals a 16 point Gaussian quadrature has been used. In table 2, we have compared our values

Table 2. Excitation total cross-sections $2s$ and $2p$ levels of Hydrogen (in units of πa_0^2) in Born, Ochkur and Glauber approximations.

Excited states	E(eV)	30	40	60	100	200
	Born	—	—	—	0 057	0.029
$2s$	Ochkur	0.123	0.106	0.081	0 053	0.028
	Glauber	0.080	0.081	0.072	0.0515	0.028
	Born	—	—	—	0.73	0.47
$2p$	Ochkur	1 12	1.09	0.95	0.73	0 48
	Glauber	0.796	0 843	0.787	0.637	0.462

for the total cross-sections for both the cases at different incident energies with the corresponding values obtained by using Ochkur (1969) and Born approximations. In figures 3 and 4, we have shown our results for the total cross-

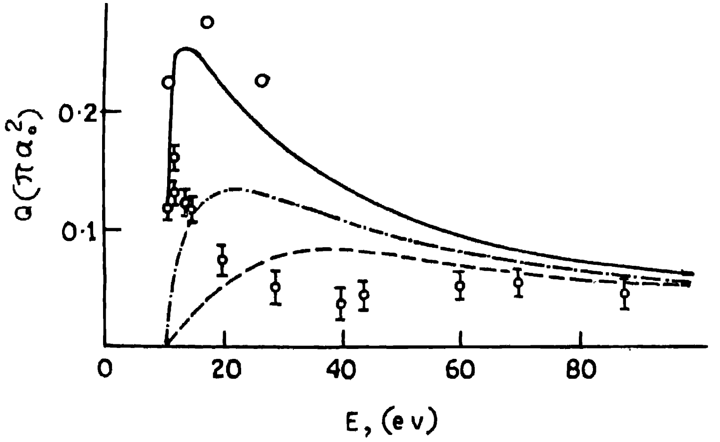

FIGURE 3. Total excitation cross-sections of the hydrogenic $2s$ level in eH scattering. Solid line—Born approximation without exchange; small circles—close coupling approximation (Damburg *et al* 1962) with consideration of $1s$–$2s$–$2p$–levels; Chained curve—Ochkur approximation (1969); dotted lines—present calculation; circles with error indicated—experiment.

sections of 1*s*–2*s* and 1*s*–2*p* processes, respectively, together with the corresponding existing theoretical curves and compared them with the experimental findings (Fite *et al* 1959, 1960, Lichten 1961).

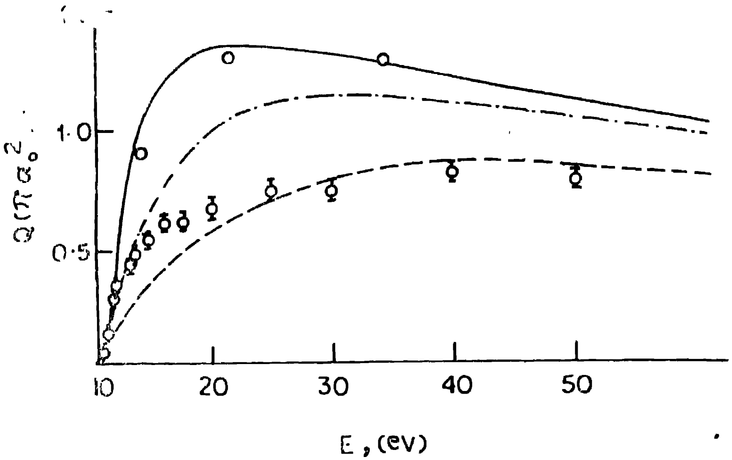

FIGURE 4. Total excitation cross-sections of the hydrogenic 2*p* level in e-H scattering. Solid line— Born approximation without exchange; small circles—close coupling approximation (Damburg *et al.* 1962) with consideration of 1-2*s*-2*p*- levels; Chained curve- -Ochkur approximation (1969), dotted line—present calculation, circles with error indicated- -experiment.

The differential cross sections obtained in Glauber approximation are more sharply peaked in the forward direction at every energy than those given by the first Born approximation. With the increase of the incident energy, our results give closer agreement with those of the first Born approximation. This is in conformity with the observation of Tai *et al.* (1969)

The calculated values for the total cross section for 1*s*–2*s* excitation agree more closely with the experimental findings than the results of other theoretical calculations in the intermediate energy range However, near the threshold energy, our results deviate considerably from the experimental findings. It does not reproduce the peak at the threshold. The theoretical curve for the total cross-section for 1*s*–2*p* excitation in Glauber approximation almost coincides with the experimental observations even upto energies as low as 25 eV; below this energy, however, there is a slight discrepancy. For both the cases under consideration, it appears from the table 2 that above 200 eV the theoretical results in first Born, Ochkur (1969) and Glauber approximations are almost identical. This has already been pointed out for the case of $1s$–$2s$ excitation in our previous note (1969). The better agreement of our theoretical results with the experiments in the intermediate energy region may be attributed to the fact that contrary to the first Born and Ochkur approximations, the Glauber approximation takes account of the interaction of the incident electron with the proton. Glauber approximation is a high energy approximation and is supposed to hold good for the small angle scattering. It is not generally expected to give good results in the low energy region Further, for the inelastic process it does not satisfy unitarity (Glauber 1959). In the present study, we have not take into account the exchange effect, which is expected to play an important role in the low energy region. All those may be responsible for the behaviour near the threshold. In addition to these, there are many doubtful assumptions in Glauber model. In order to come to a definite conclusion regarding the validity of this method, this method should be widely used in various problems and the calculated results should be compared with experimental findings. An extension of Glauber approximation to the elastic e-He scattering is under way.

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